# Math 32.1101/6 Worksheet 0

1. Express the following in the form $x^r$, r ε Q.

   a) $\left(\sqrt[5]{x}\right)^6$ b) $\sqrt[8]{x^3}$ c) $\frac{1}{\left(\sqrt{x}\right)^5}$ d) $\frac{1}{\sqrt[3]{x^4}}$

   e) $\sqrt[4]{\sqrt[3]{x}}$ f) $\sqrt{\frac{1}{\sqrt[5]{x}}}$

2. Express the following in the form $x^r$, $r$ ε Q.

   a) $x^{5/2}x^3$ b) $\frac{x^{6/7}}{x^4}$ c) $\left(x^3\right)^{-4/5}$ d) $x^{7/5}x^{-8/3}$

   e) $\left(x^{2/3}\right)^{\frac{4}{9}}$ f) $\frac{1}{x^{5/2}}$ g) $\left(\frac{1}{x^3}\right)^{-2/3}$ h) $\frac{1}{x\sqrt{x}}$

   i) $x^2\left(\sqrt[3]{x}\right)$ j) $\frac{x}{x^{2/5}}$ k) $\frac{x^{1/3}}{x}$ l) $\frac{1}{x^{-5/4}}$

3. Which of the following equations are true for all non-zero values of $x$ and $y$?

   a) $x^5+x^2=x^7$ b) $x^4x^7=x^{11}$ c) $(xy)^3=x^3y^3$

   d) $\left(x^4\right)^3=x^7$ e) $(x+y)^4=x^4+y^4$ f) $\left(x^3\right)^3=x^9$

   g) $x^7-x^3=x^4$ h) $\frac{x^8}{x^2}=x^4$ i) $\frac{x^4}{y^4}=\left(\frac{x}{y}\right)^4$

   j) $\frac{x^9}{x^3}=x^6$ k) $(x-y)^4=\frac{x^4}{y^4}$ l) $\frac{x^{10}}{y^2}=\left(\frac{x}{y}\right)^5$

4. Express the following as single polynomials:

   a) $(3x^3-4x^2+5x-7)+(5x^3+4x^2-8x+9)$

   b) $-(9x^3-4x^2-6x+10)$

   c) $(2x^5-x^3+4x^2-x)-(2x^5+x^4-x^3+2x^2-7)$

   d) $5x^2-3x-\{3x^3+4x-[8x^2-2x-(7x^2-4)]\}$

   e) $(x^3+2x^2-x-4)(x^4-2x^3+x^2-5)$

   f) $(x+1)(x+2)(x-3)-(x^2-1)(x-2)$

   g) $(x^2+2)^3-(3x^3-4)^2$

5. Express the following as single polynomials:

a) $(2x-3)^2$ b) $(x+5)^3$ c) $(2x-3)^3$

d) $(x-2)^5$ e) $(2x+1)^4$

6. Factor the following as completely as possible (using real numbers):

a) $x^4-9x^2$ b) $x^2-10$ c) $21x^2-3x^4$

d) $3x^5-24x^3$ e) $7x^6+28x^4$ f) $x^2-17x+16$

g) $-4x^4+20x^3+24x^2$ h) $6x^2-17x+7$ i) $-18x^3+36x^2-16x$

j) $x^4-81$ k) $x^4-9$ l) $x^4+5x^2-36$

m) $3x^5-18x^3+15x$ n) $-12x^6-46x^4-14x^2$ o) $x^3+8$

p) $8x^3-27$ q) $x^3+3$ r) $64x^8+27x^5$

s) $4x^2-3x^5$

7. Reduce the following as much as possible:

a) $\dfrac{x^2+6x+9}{x^2-9}$ b) $\dfrac{x^2}{4x^2+7x}$ c) $\dfrac{x^2+5x+6}{x^2+5x}$

d) $\dfrac{x^3+8}{x^2-4}$ e) $\dfrac{x^4+3x^3-10x^2}{4x^3-x^5}$

8. Express the following as rational terms with denominators of lowest possible degree:

a) $\left(\dfrac{x^2-4}{x^2+x}\right)\left(\dfrac{x+1}{x+2}\right)$ b) $\left(\dfrac{4x^3-3x^2-10x}{2x^2+12x+18}\right)\left(\dfrac{2x^2+4x-6}{3x^4-12x^3+12x^2}\right)$

c) $\left(\dfrac{x^2+5x+4}{x^2-4}\right)\div\left(\dfrac{x^2-1}{x+2}\right)$ d) $\left(\dfrac{4x^2-12x+9}{2x^2-5x+2}\right)\div\left(\dfrac{9-4x^2}{4x^2-1}\right)$

e) $\dfrac{x}{x+3}+\dfrac{5x^2}{x^2-9}$ f) $\dfrac{x}{x^2-2x+5}+\dfrac{3}{x-1}$

g) $\dfrac{x+1}{x^3-5x^2+6x}-\dfrac{x-2}{x^4-x^3-6x^2}$ h) $\dfrac{x}{x+2}+\dfrac{1}{x}-\dfrac{4}{x+1}$

i) $1-\dfrac{1}{x+1}+\dfrac{2}{x^2-1}$ j) $\dfrac{\dfrac{x}{x-2}-\dfrac{2x}{x^2-4}}{\dfrac{x-1}{x^2-9}+\dfrac{x}{x^2+5x+6}}$

k) $\dfrac{5}{x^2+x-12}+\dfrac{4}{x^2-x-20}-\dfrac{3}{x^2-8x+15}$

9. Rationalize the denominators of the following:

a) $\frac{5}{\sqrt{3}}$ b) $\frac{\sqrt{2}}{4\sqrt{7}}$ c) $\frac{1-\sqrt{3}}{2+\sqrt{3}}$

d) $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ e) $\frac{5}{x\sqrt{x}}$ f) $\frac{-7x^3}{\sqrt{x+1}}$

g) $\frac{2x}{x+\sqrt{x+2}}$ h) $\frac{7}{\sqrt{x+2}-\sqrt{x+1}}$ i) $\frac{x^2}{\sqrt{x^2-1}+\sqrt{x+3}}$

j) $\frac{\sqrt{x}}{\sqrt{x}-2}$ k) $\frac{\sqrt{x}}{\sqrt{x-2}}$

10. Find all the real solutions to the following equations:

a) $3x^2-7x+1=2x^2-2x-5$ b) $x^2-2x-2=0$

c) $x^2-2x+2=0$ d) $9x^2-30x+25=0$

e) $x^3+27=0$ f) $\frac{2}{x}-\frac{6}{x+1}=\frac{-5}{x+3}$

g) $\frac{7}{x+2}-\frac{2}{x^2-4}=1$ h) $\sqrt{3x+1}+1=x$

i) $\sqrt{3x+1}=\sqrt{x}+3$

11. Solve the following systems of equations:

a) $\begin{cases} 3x-y=-1 \\ 3x-5y=-11 \end{cases}$ b) $\begin{cases} 4x^2+y^2=68 \\ 2x+y=10 \end{cases}$

c) $\begin{cases} x^2+xy=-1 \\ x-y+3=0 \end{cases}$ d) $\begin{cases} y=x^2 \\ x+2y=2 \end{cases}$

e) $\begin{cases} y=x \\ y=x^3 \end{cases}$

## Solutions

1. a) $x^{6/5}$ b) $x^{3/8}$ c) $x^{-5/2}$ d) $x^{-4/3}$
   e) $x^{1/12}$ f) $x^{-1/10}$

2. a) $x^{11/2}$ b) $x^{-22/7}$ c) $x^{-12/5}$ d) $x^{-19/15}$
   e) $x^{8/27}$ f) $x^{-5/2}$ g) $x^{2}$ h) $x^{-3/2}$
   i) $x^{7/3}$ j) $x^{3/5}$ k) $x^{-2/3}$ l) $x^{5/4}$

3. The true solutions are b), c), f), i), and j).

4. a) $8x^3-3x+2$ b) $-9x^3+4x^2+6x-10$
   c) $-x^4+2x^2-x+7$ d) $-3x^3+6x^2-9x+4$
   e) $x^7-4x^5+2x^3-14x^2+5x+20$ f) $2x^2-6x-8$
   g) $-8x^6+6x^4+24x^3+12x^2-8$

5. a) $4x^2-12x+9$ b) $x^3+15x^2+75x+125$
   c) $8x^3-36x^2+54x-27$ d) $x^5-10x^4+40x^3-80x^2+80x-32$
   e) $16x^4+32x^3+24x^2+8x+1$

6. a) $x^2(x+3)(x-3)$ b) $\left(x+\sqrt{10}\right)\left(x-\sqrt{10}\right)$
   c) $-3x^2\left(x+\sqrt{7}\right)\left(x-\sqrt{7}\right)$ d) $3x^3\left(x+2\sqrt{2}\right)\left(x-2\sqrt{2}\right)$
   e) $7x^4(x^2+4)$ f) $(x-16)(x-1)$
   g) $-4x^2(x+1)(x-6)$ h) $(3x-7)(2x-1)$
   i) $-2x(3x-4)(3x-2)$ j) $(x^2+9)(x+3)(x-3)$
   k) $\left(x^2+3\right)\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$ l) $(x^2+9)(x+2)(x-2)$
   m) $3x(x+1)(x-1)\left(x+\sqrt{5}\right)\left(x-\sqrt{5}\right)$ n) $-2x^2(3x^2+1)(2x^2+7)$
   o) $(x+2)(x^2-2x+4)$ p) $(2x-3)(4x^2+6x+9)$
   q) $\left(x+\sqrt[3]{3}\right)\left(x^2-\sqrt[3]{3}x+\left(\sqrt[3]{3}\right)^2\right)$ r) $x^5(4x+3)(16x^2-12x+9)$
   s) $-3x^2\left(x-\sqrt[3]{\frac{4}{3}}\right)\left(x^2+\sqrt[3]{\frac{4}{3}}x+\left(\sqrt[3]{\frac{4}{3}}\right)^2\right)$

7. a) $\frac{x+3}{x-3}$ $(x \neq -3)$ b) $\frac{x}{4x+7}$ $(x \neq 0)$

c) $\frac{x^2+5x+6}{x^2+5x}$ d) $\frac{x^2-2x+4}{x-2}$ $(x \neq -2)$

e) $\frac{-x-5}{x^2+2x}$ $(x \neq 2, x \neq 0)$

8. a) $\frac{x-2}{x}$ b) $\frac{4x^2+x-5}{3x^3+3x^2-18x}$ c) $\frac{x+4}{x^2-3x+2}$

d) $\frac{-4x^2+4x+3}{2x^2-x-6}$ e) $\frac{6x^2-3x}{x^2-9}$ f) $\frac{4x^2-7x+15}{x^3-3x^2+7x-5}$

g) $\frac{x^3+2x^2+6x-4}{x^5-3x^4-4x^3+12x^2}$ h) $\frac{x^3-2x^2-5x+2}{x^3+3x^2+2x}$ i) $\frac{x^2-x+2}{x^2-1}$

j) $\frac{x^4-9x^2}{2x^3-6x^2+2x+4}$ k) $\frac{6x-49}{x^3-4x^2-17x+60}$

9. a) $\frac{5\sqrt{3}}{3}$ b) $\frac{\sqrt{14}}{28}$ c) $5-3\sqrt{3}$

d) $\frac{\sqrt{15}+3}{2}$ e) $\frac{5\sqrt{x}}{x^2}$ f) $\frac{-7x^3\sqrt{x+1}}{x+1}$

g) $\frac{2x^2-2x\sqrt{x+2}}{x^2-x-2}$ h) $7\sqrt{x+2}+7\sqrt{x+1}$ i) $\frac{x^2\sqrt{x^2-1}-x^2\sqrt{x+3}}{x^2-x-4}$

j) $\frac{x+2\sqrt{x}}{x-4}$ k) $\frac{\sqrt{x^2-2x}}{x-2}$

10. a) $\{2,3\}$ b) $\{1+\sqrt{3}, 1-\sqrt{3}\}$ c) $\phi$

d) $\{5/3\}$ e) $\{-3\}$ f) $\{2,3\}$

g) $\{3,4\}$ h) $\{5\}$ i) $\{16\}$

11. a) $\{(1/2, 5/2)\}$ b) $\{(4,2),(1,8)\}$ c) $\{(-1,2),(-1/2, 5/2)\}$

d) $\left\{\left(-1/4+\sqrt{17}/4, 9/8-\sqrt{17}/8\right), \left(-1/4-\sqrt{17}/4, 9/8+\sqrt{17}/8\right)\right\}$

e) $\{(0,0),(1,1),(-1,-1)\}$