

Part A: For each of the following terms construct its sign diagram and find for which values of  $x$ , the term is  $> 0$ ,  $< 0$ ,  $= 0$  or undefined.

- (1)  $x^2 + x - 6$       (2)  $12x^2 + 13x - 14$       (3)  $3 + 5x - 2x^2$       (4)  $(x + 5)(x^2 - 5x + 7)$   
 (5)  $\frac{(x-2)(x+1)}{x+3}$       (6)  $\frac{2x}{x^2+1}$       (7)  $x^3 - 2x^2 - 2x$       (8)  $x(x-2)^2(x+2)^3(x-4)^4$   
 (9)  $\frac{2x^2-3x}{x-2}$       (10)  $\frac{(x+3)(x^2-5)}{(x+1)^2x^3}$       (11)  $x^3 - 8$       (12)  $x^2(2x+3)^2(x-1)^2$   
 (13)  $\frac{x}{4} + \frac{1}{x^2}$       (14)  $\frac{-x^2-1}{(2x-3)(2x+5)^2}$       (15)  $3x^2 + 5x + 4$   
 (16)  $\frac{(x^2-3x+7)(x^2+4x+1)}{(-x^2+3x-5)(x^2-1)^2}$       (17)  $\frac{(x^2-9x+20)^2(x^2+2)}{(3x-x^2-7)^2(x+5)^2}$   
 (18)  $\frac{(x^4+9x^2)(x^2-6x+5)^2}{(3x-x^2-7)(x^2-49)^3}$       (19)  $\frac{(8-27x^3)(x+1)^2(2-x)^4}{(x^2-x-6)(x^2+3x+7)(6-7x)}$

Part B: Solve the following inequalities.

- (1) (a)  $x^2 < 1$       (b)  $x^2 + x + 1 > 0$       (c)  $x^3 + x \leq 0$       (d)  $6x^2 - x + 1 \leq 0$   
 (e)  $5x + 2x^2 \leq 3$       (f)  $x(3x+5) \leq 2$       (g)  $x^4 < x^3 + 6x^2$       (h)  $x + \frac{15}{x} \leq 8$   
 (i)  $\frac{x+1}{3x} \geq -1$       (j)  $x+5 < \frac{2x+3}{x-1}$       (k)  $\frac{7x-1}{x+2} \leq -2$       (l)  $\frac{2x+1}{x-1} > \frac{1}{x+1}$   
 (m)  $\frac{2}{x-3} \geq \frac{x+1}{x-4}$       (n)  $\frac{6x}{x+2} < \frac{x+1}{2x-1}$       (o)  $\frac{(2-x^2)^3}{x^3-x} > 0$
- (2) (a)  $\left\{ \begin{array}{l} 2x-7 \geq 0 \\ 5-3x < 0 \end{array} \right\}$       (b)  $\frac{1}{4-x} \geq \frac{1}{x} \geq \frac{1}{5}$       (c)  $2x+3 \leq x+7 \leq 3x+5$   
 (d)  $3 < \frac{2x+1}{x+3} < 5$       (e)  $\left\{ \begin{array}{l} \frac{x^2+2x}{x-1} \geq x-1 \\ \frac{x-1}{x+3} \leq 1 \end{array} \right\}$
- (3) (a)  $|2x+5| < 3$       (b)  $|3-7x| \leq 6$       (c)  $\left| \frac{x+3}{x-3} \right| \leq 2$   
 (d)  $\left| \frac{3-2x}{2+x} \right| < 4$       (e)  $|2x-5| < 3x+2$       (f)  $|4x+7| \leq x-2$

## Answers

### Part A:

- (1)  $> 0$  at  $(-\infty, -3) \cup (2, \infty)$ ;  $< 0$  on  $(-3, 2)$ ;  $= 0$  at  $-3, 2$
- (2)  $> 0$  on  $(-\infty, -\frac{7}{4}) \cup (\frac{2}{3}, \infty)$ ;  $< 0$  on  $(-\frac{7}{4}, \frac{2}{3})$ ;  $= 0$  at  $-\frac{7}{4}, \frac{2}{3}$
- (3)  $> 0$  on  $(-\frac{1}{2}, 3)$ ;  $< 0$  on  $(-\infty, -\frac{1}{2}) \cup (3, \infty)$ ;  $= 0$  at  $-\frac{1}{2}, 3$
- (4)  $> 0$  on  $(-5, \infty)$ ;  $< 0$  on  $(-\infty, -5)$ ;  $= 0$  at  $-5$
- (5)  $> 0$  on  $(-3, -1) \cup (2, \infty)$ ;  $< 0$  on  $(-\infty, -3) \cup (-1, 2)$ ;  $= 0$  at  $-1, 2$ ; undefined at  $-3$
- (6)  $> 0$  on  $(0, \infty)$ ;  $< 0$  on  $(-\infty, 0)$ ;  $= 0$  at  $0$
- (7)  $> 0$  on  $(1 - \sqrt{3}, 0) \cup (1 + \sqrt{3}, \infty)$ ;  $< 0$  on  $(-\infty, 1 - \sqrt{3}) \cup (0, 1 + \sqrt{3})$ ;  
 $= 0$  at  $1 - \sqrt{3}, 0, 1 + \sqrt{3}$
- (8)  $> 0$  on  $(-\infty, -2) \cup (0, 2) \cup (2, 4) \cup (4, \infty)$ ;  $< 0$  on  $(-2, 0)$ ;  $= 0$  at  $-2, 0, 2, 4$
- (9)  $> 0$  on  $(0, \frac{3}{2}) \cup (2, \infty)$ ;  $< 0$  on  $(-\infty, 0) \cup (\frac{3}{2}, 2)$ ;  $= 0$  at  $0, \frac{3}{2}$ ; undefined at  $2$
- (10)  $> 0$  on  $(-\infty, -3) \cup (-\sqrt{5}, -1) \cup (-1, 0) \cup (\sqrt{5}, \infty)$ ;  $< 0$  on  $(-3, -\sqrt{5}) \cup (0, \sqrt{5})$ ;  
 $= 0$  at  $-3, -\sqrt{5}, \sqrt{5}$ ; undefined at  $-1, 0$
- (11)  $> 0$  on  $(2, \infty)$ ;  $< 0$  on  $(-\infty, 2)$ ;  $= 0$  at  $2$
- (12)  $> 0$  on  $(-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, 0) \cup (0, 1) \cup (1, \infty)$ ;  $= 0$  at  $-\frac{3}{2}, 0, 1$
- (13)  $> 0$  on  $(-\sqrt[3]{4}, 0) \cup (0, \infty)$ ;  $< 0$  on  $(-\infty, -\sqrt[3]{4})$ ;  $= 0$  at  $-\sqrt[3]{4}$ ; undefined at  $0$
- (14)  $> 0$  on  $(-\infty, -\frac{5}{2}) \cup (-\frac{5}{2}, \frac{3}{2})$ ;  $< 0$  on  $(\frac{3}{2}, \infty)$ ; undefined at  $-\frac{5}{2}, \frac{3}{2}$
- (15)  $> 0$  on  $(-\infty, \infty)$
- (16)  $> 0$  on  $(-2 - \sqrt{3}, -1) \cup (-1, -2 + \sqrt{3})$ ;  $< 0$  on  $(-\infty, -2 - \sqrt{3}) \cup (-2 + \sqrt{3}, 1) \cup (1, \infty)$ ;  
 $= 0$  at  $-2 - \sqrt{3}, -2 + \sqrt{3}$ ; undefined at  $-1, 1$
- (17)  $> 0$  on  $(-\infty, -5) \cup (-5, 4) \cup (4, 5) \cup (5, \infty)$ ;  $= 0$  at  $4, 5$ ; undefined at  $-5$
- (18)  $> 0$  on  $(-7, 0) \cup (0, 1) \cup (1, 5) \cup (5, 7)$ ;  $< 0$  on  $(-\infty, -7) \cup (7, \infty)$ ;  $= 0$  at  $0, 1, 5$ ;  
 undefined at  $-7, 7$
- (19)  $> 0$  on  $(-\infty, -2) \cup (\frac{2}{3}, \frac{6}{7}) \cup (3, \infty)$ ;  $< 0$  on  $(-2, -1) \cup (-1, \frac{2}{3}) \cup (\frac{6}{7}, 2) \cup (2, 3)$ ;  
 $= 0$  at  $-1, \frac{2}{3}, 2$ ; undefined at  $-2, \frac{6}{7}, 3$

### Part B:

- (1) (a)  $(-1, 1)$  (b)  $(-\infty, \infty)$  (c)  $(-\infty, 0]$  (d)  $\emptyset$  (e)  $[-3, \frac{1}{2}]$  (f)  $[-2, \frac{1}{3}]$   
 (g)  $(-2, 0) \cup (0, 3)$  (h)  $(-\infty, 0) \cup [3, 5]$  (i)  $(-\infty, -\frac{1}{4}] \cup (0, \infty)$   
 (j)  $(-\infty, -4) \cup (1, 2)$  (k)  $(-2, -\frac{1}{3}]$  (l)  $(-\infty, -1) \cup (1, \infty)$  (m)  $(3, 4)$   
 (n)  $(-2, -\frac{2}{11}) \cup (\frac{1}{2}, 1)$  (o)  $(-\infty, -\sqrt{2}) \cup (-1, 0) \cup (1, \sqrt{2})$
- (2) (a)  $[\frac{7}{2}, \infty)$  (b)  $[2, 4)$  (c)  $[1, 4]$  (d)  $(-8, -\frac{14}{3})$  (e)  $(-3, \frac{1}{4}] \cup (1, \infty)$
- (3) (a)  $(-4, -1)$  (b)  $[-\frac{3}{7}, \frac{9}{7}]$  (c)  $(-\infty, 1] \cup [9, \infty)$  (d)  $(-\infty, -\frac{11}{2}) \cup (-\frac{5}{6}, \infty)$   
 (e)  $(\frac{3}{5}, \infty)$  (f)  $\emptyset$