MATH-1101 (6) Intro to Calculus – Final Examination

April 13, 2009 - 9:00 a.m. **Duration:** 3 hours
Instructor: A. Bendor-Samuel, B. Bector, M. Nielsen, H. Tram, K. Livesley

STUDENT NAME: _____ STUDENT NO.: _____

Special Instructions:

WRITE YOUR NAME IN THE SPACE ABOVE AND CIRCLE YOUR INSTRUCTOR'S NAME. There are sixteen questions. Answer all of the questions on these pages in the spaces following each question. Use the backs of pages for rough work if necessary. Calculators or similar electronic aids are **NOT permitted.**
The following table is for the marker's use.

1	2	3	4	5	6	7	8	
9	10	11	12	13	14	15		Total

Value	
17	1. Differentiate the following: **[Do not simplify]** (a) $f(x) = 3^x - \dfrac{3}{\sqrt{x}} + \log_3 x^2 + \tan\left(\dfrac{\pi}{4}\right) - \dfrac{1}{x}$

(b) $f(x) = \left(\sin^{-1}(x^2) + e^x\right)\left(x^3 + \ln x\right)$

Value

Differentiate the following: **[Do not simplify]**

(c) $f(x) = \sec^4(\sin(e^x))$

(d) $f(x) = \dfrac{\tan^{-1}(x^2+1)}{x^3+e^{x^2}}$

(e) $f(x) = (\sin 2x)^x$

Value	
	(f) $(x) = \int_2^{\cos x} \left(2t^3 + t - 1\right)\, dt$
4	2. Find the equation of the tangent line to the curve $y \ln x - ye^x + x = 1$ at the point $(1, 0)$.

Value	
24	3. Evaluate each of the following integrals:

(a) $\int \left(3x^2 + e^{3x} + 3^x - \frac{3}{x} + \pi\right) dx$

(b) $\int_0^1 \frac{x}{\sqrt[3]{x^2+5}}\, dx$

(c) $\int \cos^4 x\, dx$

Value

Evaluate each of the following integrals:

(d) $\int x \cot^{-1}(x)\, dx$

(e) $\int \frac{5x^2 - 3x + 7}{(x-1)(x^2+2)}\, dx$

Value	

Evaluate the following integral:

(f) $\displaystyle\int \frac{x}{\sqrt{8x - x^2}}\, dx$

Value	
3	4. Find the area of the region bounded by the curves $y^2 = x$ and $y - x + 6 = 0$.

Value
8

5. Given that $f(x) = \dfrac{3x^2+9x}{(x+1)^2}$, $f'(x) = \dfrac{-3x+9}{(x+1)^3}$ and $f''(x) = \dfrac{6(x-5)}{(x+1)^4}$,

   (a) Calculate $\lim\limits_{x\to -1^-} f(x)$, $\lim\limits_{x\to -1^+} f(x)$, $\lim\limits_{x\to \infty} f(x)$ and $\lim\limits_{x\to -\infty} f(x)$.

   (b) Construct sign diagrams for $f'(x)$ and $f''(x)$. Identify the intervals where $f(x)$ is increasing/decreasing and concave up/concave down, using interval notation.

   (c) Sketch the graph of f. Label all intercepts, local extreme values, all points of inflection, and any horizontal and vertical asymptotes.

(next page is left blank if more space is required.)

Value	

(Use this page for Question 5)

Value	
4	6. (a) Let $f(x)$ be a function. State the definition of the derivative $f'(x)$. (b) Use the definition of the derivative to find the derivative of $f(x) = \sqrt{2-3x}$.

Value	
5	7. A long ladder projects over the top of a wall 12 feet high. If the lower end of the ladder is pulled away from the wall at 3 ft. per second, how fast is the angle between the wall and the ladder changing, when the base of the ladder is 5 ft. from the wall?

Value	
5	8. A rectangular sheet of metal has a perimeter of 30 cm, and is to be rolled into a cylinder open at both ends. Find the dimensions of the rectangle that give the cylinder of maximum volume.

Value	
7	9. Evaluate the limits. Do NOT use l'Hospital's Rule.

(a) $\lim_{x \to 1} \dfrac{-x^3 + x^2 - 4x + 4}{x^2 - 2x + 1}$

(b) $\lim_{x \to 0} \dfrac{\sin^2 x}{x \sin 3x}$

(c) $\lim_{x \to -\infty} \dfrac{\sqrt{9x^6 - x}}{x^3 + 1}$

Value	
5	10. Evaluate the limit. [You may use l'Hospital's Rule.]

(a) $\lim_{x \to 0} \dfrac{x^2}{1 - \cos 2x}$

(b) $\lim_{x \to \infty} (e^x + x)^{\frac{1}{x}}$

3	11. Evaluate the integral $\displaystyle\int_1^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx$.

Value	
5	12. (a) State (but do NOT prove) the Mean Value Theorem for Derivatives.

(b) Use the Mean Value Theorem to prove that if $|f'(x)| > 1$ for every $x \in [a, b]$ then $|f(b) - f(a)| > |b - a|$.

Value	
4	13. Given $f(x) = \begin{cases} \dfrac{x^2-1}{x^2+2x-3} & x < 1 \\ \dfrac{2-x}{2x} & x \geq 1 \end{cases}$ (a) State the domain of $f(x)$. (b) Identify the interval(s) where $f(x)$ is continuous on $\mathbb{R}$.

Value	
3	14. Define what it means for f to be continuous at $x = a$. Prove that if f is differentiable at $x = a$, then f is continuous at $x = a$.
3	15. Define the inverse sine function, $\sin^{-1} x$. Show that $\dfrac{d}{dx} \sin^{-1} x = \dfrac{1}{\sqrt{1 - x^2}}$.
Total 100	