MATH-32.1101/6

Intro to Calculus – Final Examination

April 9, 2008 - 9:00 a.m. **Duration:** 3 hours

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STUDENT NAME: ____________________ STUDENT NO.: ____________________

Special Instructions:

WRITE YOUR NAME IN THE SPACE ABOVE AND CIRCLE YOUR INSTRUCTOR'S NAME. There are sixteen questions. Answer all of the questions on these pages in the spaces following each question. Use the backs of pages for rough work if necessary. Calculators or similar electronic aids are **NOT permitted.**

The following table is for the marker's use.

1	2	3	4	5	6	7	8	
9	10	11	12	13	14	15	16	Total

Value

16 1. Differentiate the following: **[Do not simplify]**

(a) $f(x) = 8x^7 - 7x^{-8} + 8^x + \dfrac{2}{\sqrt[3]{x}} - \log_5 x + \ln 2e$

(b) $f(x) = (\sec x - \csc x)(\tan x + \cot x)$

Value

Differentiate the following: [Do not simplify]

(c) $f(x) = \dfrac{e^{3x}}{\sin^{-1} x - \cos x}$

(d) $f(x) = \ln\left(\sin^2(3x+2) + 1\right)$

(e) $f(x) = (\tan^{-1} x)^{\sqrt{x}}$

Value	
3	2. Find the equation of the tangent line to the curve $\sin x + \cos y = \sin x \cos y + \left(\sqrt{2} - \frac{1}{2}\right)$ at the point $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$.

Value

24

3. Find each of the following integrals:

(a) $\int \left(e^{2x} + 2^x + x^2 + \frac{2}{x} + 2 \right) dx$

(b) $\int_0^{\pi/4} \frac{\sec^2 x}{1 + \tan x} \, dx$

(c) $\int \sin^{-1} x \, dx$

Value

Find each of the following integrals:

(d) $\int \sin^2 x \cos^2 x \, dx$

(e) $\int \frac{\sqrt{x^2 - 1}}{x} \, dx$

Value

Find the following integrals:

(f) $\displaystyle\int \frac{dx}{\sqrt{7+6x-x^2}}$

(g) $\displaystyle\int \frac{2x^2-3x+1}{x^3+x}\,dx$

Value	
3	4. Find the area of the region bounded by the curves $y = x^3$ and $y = 4x$.

Value	
8	5. Given that $f(x) = \dfrac{9(x-1)(x+3)}{(x-3)^2}$, $f'(x) = \dfrac{-72x}{(x-3)^3}$ and $f''(x) = \dfrac{72(2x+3)}{(x-3)^4}$,

(a) Calculate $\lim_{x \to 3^-} f(x)$, $\lim_{x \to 3+} f(x)$, $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

(b) Construct sign diagrams for $f'(x)$ and $f''(x)$. Identify the intervals where $f(x)$ is increasing/decreasing and concave up/concave down, using interval notation.

(c) Sketch the graph of f. Label all intercepts, local extreme values, all points of inflection, and any horizontal and vertical asymptotes.

Value

(Use this page for Question 5)

Value	
4	6. (a) Let $f(x)$ be a function. State the definition of the derivative $f'(x)$.
	(b) Use the definition of the derivative to find the derivative of $f(x) = \frac{1}{3-2x}$.

Value	
5	7. 36π cm^3 of clay is being shaped on a pottery wheel. The clay is in the shape of a right circular cylinder. If the radius of the cylinder is decreasing at 1 cm per second, how fast is the height changing when the radius is 3 cm?
5	8. A box has a square base and no top. The total area of the sides and base of the box is 48 cm^2. Find the dimensions of the box so it has maximum volume.

Value	
	9. Evaluate the limits. Do NOT use l'Hospital's Rule.
2	(a) $\lim_{x \to 1} \dfrac{\sqrt{x^2+3}-2}{x^2-1}$
2	(b) $\lim_{x \to 0} \dfrac{1-\cos x}{x^2}$
2	(c) $\lim_{x \to 0} x^2 \sin\left(\dfrac{1}{x}\right)$

Value

2 10. Evaluate the limit. [You may use l'Hospital's Rule.]

(a) $\lim_{x \to 0} \frac{\sin^{-1}(2x)}{x}$

2 (b) $\lim_{x \to 0^+} x \ln(\sin x)$

2 11. (a) State both parts of the Fundamental Theorem of Calculus.

Value	
2	(b) Let $y = \int_0^{x^2} \sqrt{4t^3 + 2}\, dt$. Find $\frac{dy}{dx}$.
3	12. Evaluate the integral $\int_{1/2}^{\infty} x^{-2}\, dx$.

Value	
6	13. Use the definition of the derivative to prove the following:

(a) The Product Rule. State the Product Rule before you prove it.

(b) $\frac{d}{dx}\cos x = -\sin x$.

Value	
3	14. Define what it means for f to be continuous at $x = a$. Prove that if f is differentiable at $x = a$, then f is continuous at $x = a$.
3	15. Define the inverse sine function, $\sin^{-1} x$. Show that $\dfrac{d}{dx}\sin^{-1} x = \dfrac{1}{\sqrt{1-x^2}}$.

Value	
3	16. (a) State the Mean Value Theorem for derivatives.

(b) Prove that if $f'(x) = 0$ for all $x \in (a, b)$, then f is constant on (a, b).

Total
100