MATH-1101 (6) Intro to Calculus – Final Examination

June 7, 2013 **Duration:** 3 hours

STUDENT NAME: Answer Key STUDENT NO.: ____________

Special Instructions:

WRITE YOUR NAME IN THE SPACE ABOVE AND CIRCLE YOUR INSTRUCTOR'S NAME. There are fifteen questions. Answer all of the questions on these pages in the spaces following each question. Use the backs of pages for rough work if necessary. Calculators or similar electronic aids are **NOT permitted.**

The following table is for the marker's use.

1	2	3	4	5	6	7	8	
9	10	11	12	13	14	15		Total

Value

17 1. Differentiate the following: **[Do not simplify]**

(a) $f(x) = \sec(3x) + 7\sqrt[5]{x^3} + \pi^x + \frac{\pi}{x} + \sin\left(3\frac{\pi}{2}\right)$

3

$$f' = \sec(3x)\tan(3x)(3) + 7\left(\tfrac{3}{5}\right)x^{-2/5} + \pi^x \ln\pi$$

$$+\pi(-1)x^{-2}$$

(b) $f(x) = \underbrace{[\ln(\csc(2x) - \cos x)]}\cdot\underbrace{\sin^{-1}(e^{2x})}$

3

$$f'(x) = [\ln(\csc 2x - \cos x)]\cdot\frac{1}{\sqrt{1-(e^{2x})^2}}(e^{2x})(2)$$

$$+\sin^{-1}(e^{2x})\left[\frac{1}{\csc(2x) - \cos x}\right]\left[-\csc(2x)\cot(2x)(2) + \sin x\right]$$

Value

Differentiate the following: [**Do not simplify**]

(c) $f(x) = \sin^2\left(e^{\tan(2x)} + \ln\left(\sec^{-1}(\sin 2x)\right)\right)$

3

$$f' = 2\sin\left(e^{\tan 2x} + \ln\left(\sec^{-1}(\sin 2x)\right)\right) \cdot \cos\left(e^{\tan 2x} + \ln\left(\sec^{-1}(\sin 2x)\right)\right) \cdot \left[e^{\tan 2x} \cdot \sec^2(2x) \cdot 2 + \frac{1}{\sec^{-1}(\sin(2x))} \cdot \frac{1\cos(2x)(2)}{\sin 2x\sqrt{\sin^2(2x)-1}}\right]$$

(d) $f(x) = \dfrac{\log_7 \tan^{-1}(3x)}{\csc^{-1} x + e^{x^2}}$

3

$$f' = \frac{\frac{(\csc^{-1}x + e^{x^2})}{(\tan^{-1}(3x)\ln 7)} \cdot \frac{(3)}{(3x)^2+1} - \left[\log_7 \tan^{-1}(3x)\right] \cdot \left[\frac{-1}{x\sqrt{x^2-1}} + e^{x^2} 2x\right]}{\left[\csc^{-1}x + e^{x^2}\right]^2}$$

(e) $f(x) = (2x+1)^{\sin(3x)}$ $= y$

$$\ln y = \sin 3x \ln(2x+1)$$

3

$$\frac{1}{y}\frac{dy}{dx} = \frac{(\sin 3x)(2)}{(2x+1)} + \ln(2x+1)\cos(3x)(3)$$

$$\therefore \frac{dy}{dx} = (2x+1)^{\sin 3x}\left[\frac{(\sin 3x)(2)}{2x+1} + \ln(2x+1)\cos(3x)(3)\right]$$

Value

Differentiate the following: **[Do not simplify]**

(f) $f(x) = \int_{\cos x}^{\pi} \sin(1+3t^2)\,dt$

2

$f' = -\sin(1+3\cos^2 x)\cdot(-\sin x)$

4 2. Find the equation of the tangent line to the curve $y\sin(2x) = x\cos(2y)$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$.

$y\cos(2x)(2) + \sin(2x)\frac{dy}{dx} = x(-\sin(2y))2\frac{dy}{dx} + \cos 2y$

@ $(\pi/2, \pi/4)$:

$\underbrace{\frac{\pi}{4}\cos(\pi)(2)} + \cancel{\sin(\pi)}^{\,0}\frac{dy}{dx} = \frac{\pi}{2}\left(-\sin\frac{\pi}{2}\right)2\frac{dy}{dx} + \cancel{\cos\frac{\pi}{2}}^{\,0}$

$-\frac{\pi}{2} = -\pi\frac{dy}{dx}$ $\therefore m = \frac{1}{2}$

$\therefore$ tangent line is

$y - \frac{\pi}{4} = \frac{1}{2}\left(x - \frac{\pi}{2}\right)$

Value 22

3. Evaluate each of the following integrals:

(a) $\int \left(e^{-5x} - \frac{1}{x\sqrt{4x^2-5}} + \underbrace{\frac{1}{\cos(3x)}}_{\sec(3x)} + 2^x\right) dx$

4

$$\boxed{= \frac{e^{-5x}}{-5} - \frac{1}{\sqrt{5}}\sec^{-1}\left(\frac{2x}{\sqrt{5}}\right) + \frac{\ln|\sec 3x + \tan 3x|}{3} + \frac{2^x}{\ln 2} + C}$$

$\int \frac{1}{x\sqrt{4x^2-5}}\,dx$ $4x^2-5$ $\Rightarrow 5\sec^2\theta - 5$ $= 5\tan^2\theta$

$2x = \sqrt{5}\sec\theta$

$dx = \frac{\sqrt{5}}{2}\sec\theta\tan\theta\,d\theta$

$= \frac{\sqrt{5}}{2}\int \frac{\sec\theta\tan\theta}{\frac{\sqrt{5}}{2}\sec\theta\,\sqrt{5}\tan\theta}\,d\theta = \frac{1}{\sqrt{5}}\theta$

$= \frac{1}{\sqrt{5}}\sec^{-1}(2x/\sqrt{5})$

(b) $\int e^{-2x}\cos(3x)\,dx$

$u = e^{-2x}$ $dv = \cos 3x\,dx$

$du = -2e^{-2x}dx$ $v = \frac{\sin 3x}{3}$

5

$= \frac{1}{3}e^{-2x}\sin 3x + \frac{2}{3}\int \sin 3x\, e^{-2x}\,dx$ $u = e^{-2x}$ $dv = \sin 3x\,dx$

$du = -2e^{-2x}dx$ $v = \frac{-\cos 3x}{3}$

$= \frac{1}{3}e^{-2x}\sin 3x + \frac{2}{3}\left[-\frac{1}{3}\cos 3x\, e^{-2x} - \frac{2}{3}\int \cos 3x\, e^{-2x}\,dx\right]$

$\int e^{-2x}\cos 3x\,dx = \frac{1}{3}e^{-2x}\sin 3x - \frac{2}{9}e^{-2x}\cos 3x - \frac{4}{9}\int e^{-2x}\cos 3x\,dx$

$\frac{13}{9}\int e^{-2x}\cos 3x\,dx = \frac{3}{9}e^{-2x}\sin 3x - \frac{2}{9}e^{-2x}\cos 3x$

$\therefore \int e^{-2x}\cos 3x\,dx = \frac{3}{13}e^{-2x}\sin 3x - \frac{2}{13}e^{-2x}\cos 3x + C$

Value

Evaluate each of the following integrals:

(c) $\int \cos^5 x \sin^4 x \, dx$

$$= \int \cos x \,(1-\sin^2 x)(1-\sin^2 x)\sin^4 x \, dx$$

4

$$= \int \cos x \left[\sin^4 x - 2\sin^6 x + \sin^8 x\right] dx$$

$$= \frac{1}{5}\sin^5 x - \frac{2}{7}\sin^7 x + \frac{1}{9}\sin^9 x + C$$

(d) $\int \frac{x^2}{\sqrt{4-x^2}} \, dx$

$4 - x^2 \to 4 - 4\sin^2\theta = 4\cos^2\theta$

$\therefore \; x = 2\sin\theta$

$dx = 2\cos\theta \, d\theta$

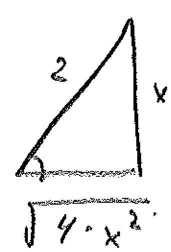
$$= \int \frac{(2\sin\theta)^2 \, 2\cos\theta}{2\cos\theta} \, d\theta$$

4

$$= 4\int \sin^2\theta \, d\theta = 4\int \frac{1-\cos 2\theta}{2} \, d\theta$$

$$= 2\left[\theta - \frac{\sin 2\theta}{2}\right] + C$$

$$= 2\left[\theta - \sin\theta\cos\theta\right] + C$$



$$= 2\left[\sin^{-1}\left(\frac{x}{2}\right) - \frac{x}{2}\,\frac{\sqrt{4-x^2}}{2}\right] + C$$

Value

Evaluate each of the following integral:

(e) $\int \frac{x^2+20x+24}{(2x-3)(x^2+9)}\,dx = \int \frac{A}{2x-3} + \frac{Bx+C}{x^2+9}\,dx$

5

$x^2+20x+24 = A(x^2+9) + (Bx+C)(2x-3)$

if $x=\frac{3}{2}$: $\frac{9}{4}+30+24 = A\left(\frac{9}{4}+9\right)$

$\frac{225}{4} = A\left(\frac{45}{4}\right) \quad A=5$

x^2 : $1 = A+2B \quad B=-2$

x^0 : $24 = 9A-3C$

$3C = 45-24 = 21 \quad C = 7$

$\int \frac{5}{2x-3} - \frac{2x}{x^2+9} + \frac{7}{x^2+9}\,dx$

$= \boxed{\frac{5\ln|2x-3|}{2} - \ln|x^2+9| + \frac{7}{3}\tan^{-1}\left(\frac{x}{3}\right) + D}$

$\int \frac{1}{x^2+9}\,dx$

x^2+9

$9\tan^2\theta+9 = 9\sec^2\theta$

$x = 3\tan\theta$

$dx = 3\sec^2\theta\,d\theta$

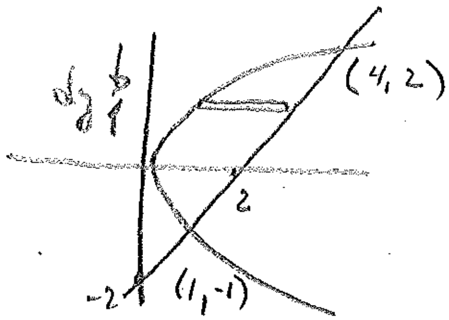
$= \int \frac{3\sec^2\theta}{9\sec^2\theta}\,d\theta$

$= \frac{1}{3}\theta = \frac{1}{3}\tan\left(x/3\right)$

Value

3 4. Given the curves, $x = y^2$ and $x - y = 2$

(a) Find the area of the region bounded by the above curves.



$$A = \int_{-1}^{2} (y + 2 - y^2)\, dy$$

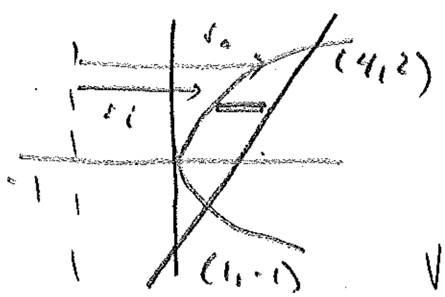
$$= \frac{y^2}{2} + 2y - \frac{y^3}{3} \Big|_{-1}^{2}$$

$$= \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$= 6 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3} = 8 - \frac{1}{2} - \frac{9}{3} = 5 - \frac{1}{2}$$

$$= 4\tfrac{1}{2}$$

3 (b) If the area bounded by the above curves is revolved about the line $x = -1$, find the volume of the solid of revolution.



$$r_o = y + 2 + 1 = y + 3$$

$$r_i = y^2 + 1$$

$$V = \pi \int_{-1}^{2} (y+3)^2 - (y^2+1)^2\, dy$$

$$V = \pi \int_{-1}^{2} \left(y^2 + 6y + 9 - y^4 - 2y^2 - 1\right) dy = \pi \int_{-1}^{2} \left(-y^4 - y^2 + 6y + 8\right) dy$$

$$= \pi \left[-\frac{1}{5}y^5 - \frac{1}{3}y^3 + 3y^2 + 8y\right]_{-1}^{2}$$

$$= \pi\left[-\frac{32}{5} - \frac{8}{3} + 12 + 16\right] - \pi\left[\frac{1}{5} + \frac{1}{3} + 3 - 8\right]$$

33 - 3

$-33/5$

30 - 33/5

$$= \frac{117\pi}{5}$$

Value

6 5. Given that $f(x) = \dfrac{(2x-1)^2}{(x+1)^2}$, $f'(x) = \dfrac{6(2x-1)}{(x+1)^3}$, $f''(x) = \dfrac{6(5-4x)}{(x+1)^4}$

(a) Construct sign diagrams for $f'(x)$ and $f''(x)$. Identify the intervals where $f(x)$ is increasing/decreasing and concave up/concave down, using interval notation.

(b) Sketch the graph of f. Label all intercepts, local extreme values, all points of inflection, and any horizontal and vertical asymptotes.

(next page is left blank if more space is required.)

a)

		-1		$\frac{1}{2}$	
$6(2x-1)$			$-$		$+$
$(x+1)^3$	$-$		$+$		
f'	$+$		$-$		$+$

		-1		$5/4$	
$6(5-4x)$			$+$		$-$
$(x+1)^4$	$+$		$+$		
f''	$+$		$+$		$-$
	$\cup$		$\cup$		$\cap$

3

$f(x)$ is

increasing on $(-\infty,-1) \cup (\frac{1}{2},\infty)$

decreasing on $(-1,\frac{1}{2})$

concave up on $(-\infty,-1) \cup (-1,5/4)$.

concave down on $(5/4,\infty)$

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intercepts: $x = \frac{1}{2}$, $y = 1$

asymptotes: $\lim\limits_{x\to\infty} \dfrac{(2x-1)^2}{(x+1)^2} = 4$

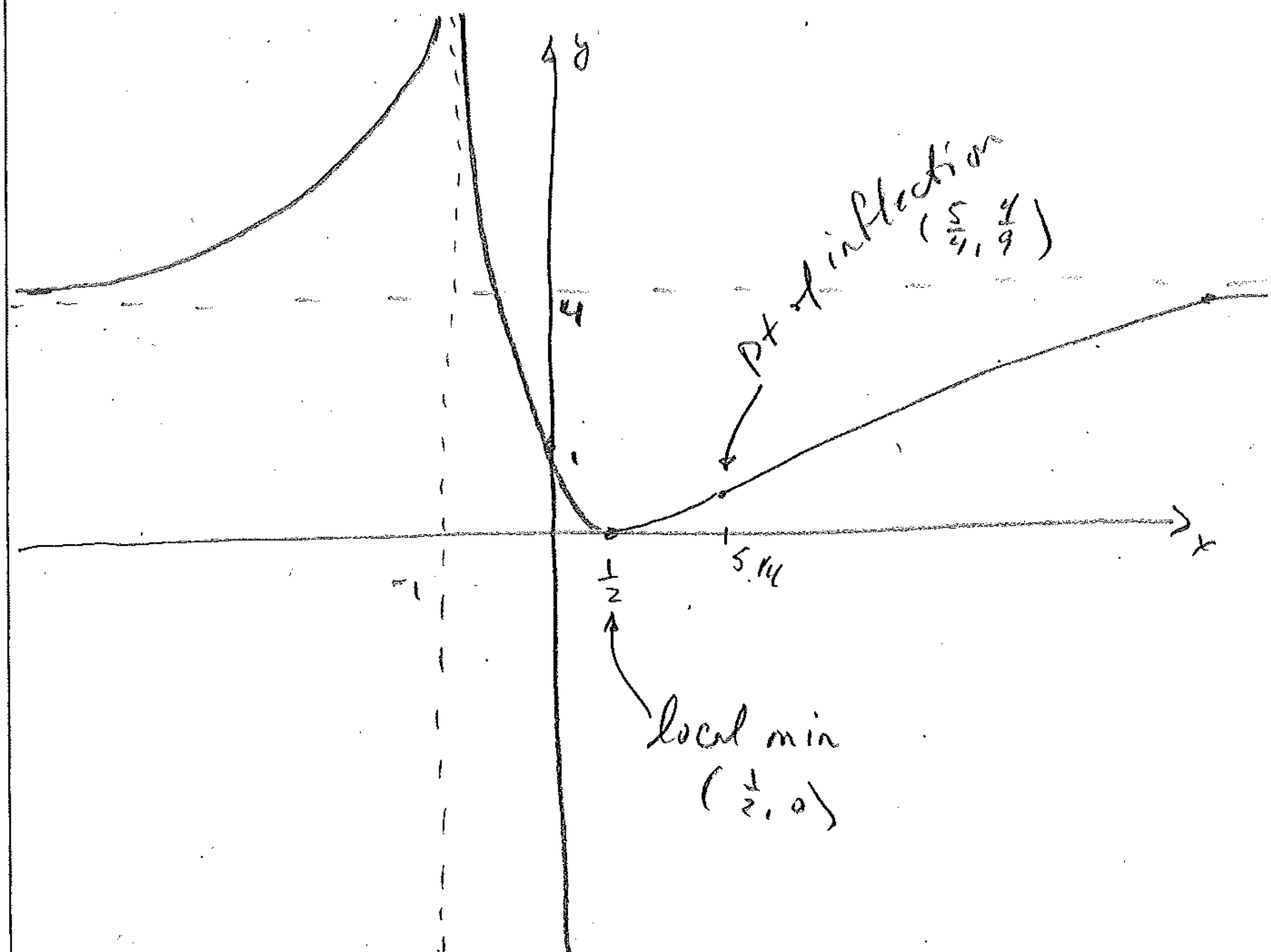
$\lim\limits_{x\to-\infty} \dfrac{(2x-1)^2}{(x+1)^2} = 4$

$\lim\limits_{x\to-1^-} \dfrac{(2x-1)^2}{(x+1)^2} = +\infty$

$\lim\limits_{x\to-1^+} \dfrac{(2x-1)^2}{(x+1)^2} = +\infty$

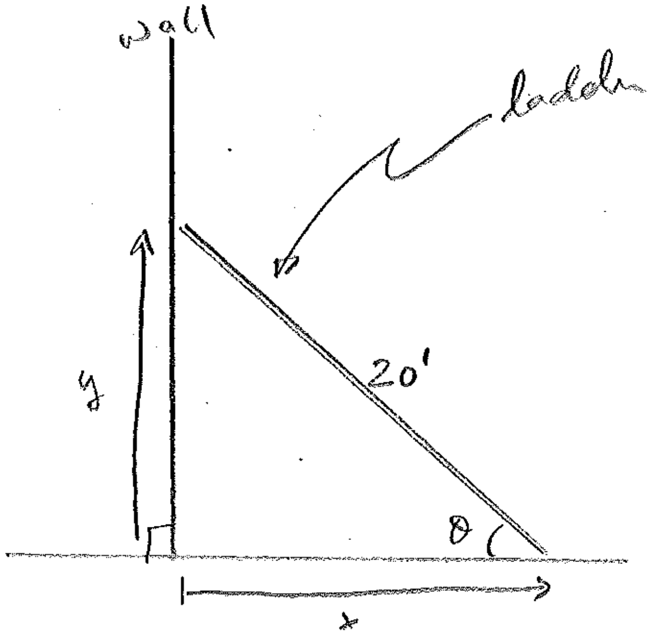
Value

(Use this page for Question 5)



3

Value	
4	6. A ladder 20 feet long leans against a vertical building. If the bottom of the ladder slides away from the building horizontally at a rate of 2 ft/sec, at what rate is the angle between the ladder and the ground changing when the top of the ladder is 12 feet above the ground?



$$\left.\frac{d\theta}{dt}\right|_{y=12} = ?$$

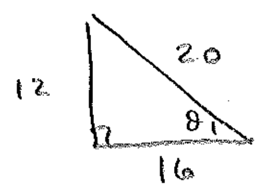
$$\frac{dx}{dt} = 2 \text{ fps}$$

$$\cos\theta = \frac{x}{20}$$

$$-\sin\theta \frac{d\theta}{dt} = \frac{1}{20}\frac{dx}{dt}$$

$$\therefore \frac{d\theta}{dt} = \frac{\frac{1}{20}\frac{dx}{dt}}{-\sin\theta}$$

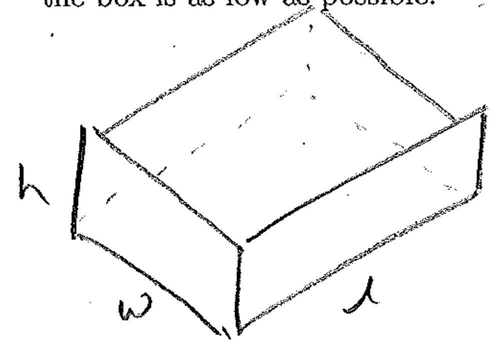
when $y = 12$



$$\sin\theta = \frac{12}{20} = \frac{3}{5}$$

$$\therefore \left.\frac{d\theta}{dt}\right|_{y=20} = \frac{\left(\frac{1}{20}\right)(2)}{-\left(3/5\right)} = -\frac{1}{10}\left(\frac{5}{3}\right) = \boxed{-\frac{1}{6} \text{ rad/sec}}$$

Value	
4	7. A box with an open top is to have a volume of $27m^3$. The length of its base is twice the width. Material for the base costs \$6 per square metre. Material for the sides costs \$1 per square metre. Find the dimensions of such box so that the total cost of constructing the box is as low as possible.



minimize Cost C

$V = 27 = w l h$

$l = 2w$

$\therefore\ 27 = 2w^2 h$

$h = \dfrac{27}{2w^2}$

$$C = 6w\overset{2w}{l} + 1(2)(wh) + 1(2)(\overset{2w}{l}h)$$

$$C = 12w^2 + 2wh + 4wh$$

$$C = 12w^2 + 6wh$$

$$C = 12w^2 + 6w\frac{27}{2w^2} = 12w^2 + \frac{81}{w}, \quad w > 0$$

$$\frac{dC}{dw} = 24w - \frac{81}{w^2} = \frac{24w^3 - 81}{w^2} = \frac{3(8w^3 - 27)}{w^2}$$

$$\frac{dC}{dw} = 0 \text{ if } w^3 = \frac{27}{8} \rightarrow w = 3/2$$

	0		3/2	
$3(8w^3-27)$		$-$		$+$
w^2			$+$	
$\frac{dC}{dw}$		$-$		$+$

min cost

Min Cost if

$w = 3/2$ m

$l = 3$ m

$h = 6$ m

Value 8

8. Evaluate the limits. Do NOT use l'Hospital's Rule.

3

(a) $\lim_{x\to -\frac{1}{2}} \left[\frac{4x^2-4x-3}{12x^3-16x^2-5x+3}\right]$

$$\begin{array}{r} 6x^2 - 11x + 3 \\ 2x+1 \enclose{longdiv}{12x^3 - 16x^2 - 5x + 3} \\ 12x^3 + 6x^2 \\ \hline -22x^2 - 5x \\ -22x^2 - 11x \\ \hline 6x + 3 \\ 6x + 3 \\ \hline 0 \end{array}$$

$$= \lim_{x\to -\frac{1}{2}} \frac{(2x-3)\cancel{(2x+1)}}{\cancel{(2x+1)}(6x^2-11x+3)}$$

$$= \frac{-4}{\frac{6}{4}+\frac{11}{2}+3} = \frac{-4}{10} = -2/5$$

3

(b) $\lim_{x\to 0} \left[\frac{x\sin(\pi x)}{\sin^2(3x)}\right]$

$$= \lim_{x\to 0} \frac{\pi \overbrace{\frac{\sin(\pi x)}{(\pi x)}}^{\to 1}}{3\ \underbrace{\frac{\sin(3x)}{3x}}_{\to 1}\ \overbrace{\frac{\sin(3x)}{x(3)}}^{\to 1}(3)} = \frac{\pi}{9}$$

2

(c) $\lim_{x\to -\infty} \frac{(5+6x-7x^3)/x^3}{(\sqrt{4x^6-7x^2-13})/x^3}$ $\longrightarrow -\sqrt{x^6}$

$$= \lim_{x\to -\infty} \frac{\cancel{\frac{5}{x^3}}^{\to 0} + \cancel{\frac{6}{x^2}}^{\to 0} - 7}{-\sqrt{4 - \cancel{\frac{7}{x^4}}_{\to 0} - \cancel{\frac{13}{x^6}}_{\to 0}}} = \frac{-7}{-2} = \frac{7}{2}$$

Value

6 9. Evaluate the limit. [You may use l'Hospital's Rule.]

(a) $\lim_{x\to 0}\left[\frac{2x-\sin(2x)}{1-\cos(2x)}\right]$ $\frac{0}{0}$

$= \lim_{x\to 0} \frac{2-2\cos 2x}{2\sin 2x} = \lim_{x\to 0} \frac{1-\cos 2x}{\sin 2x} \quad \left(\frac{0}{0}\right)$

3

$= \lim_{x\to 0} \frac{2\sin 2x}{2\cos 2x} = \frac{0}{1} = \underline{\underline{0}}$

(b) $\lim_{x\to\infty}\left[\left(1-\frac{2}{x+1}\right)^{x+1}\right] = y \qquad \to (1)^{\infty}$

$\frac{x+1-2}{x+1} = \frac{x-1}{x+1}$

3 $\therefore \ln y = \lim_{x\to\infty} (x+1)\ln\left[\frac{x-1}{x+1}\right] \to \infty(0)$

$\ln y = \lim_{x\to\infty} \frac{\ln\left[\frac{x-1}{x+1}\right]}{\frac{1}{x+1}} \quad \left(\frac{0}{0}\right)$

$\ln y = \lim_{x\to\infty} \frac{\frac{1}{\frac{x-1}{x+1}} \cdot \frac{x+1-(x-1)}{(x+1)^2}}{-\frac{1}{(x+1)^2}} =$

$\ln y = \lim_{x\to\infty} \frac{-(x+1)(2)}{(x-1)} = -2$

$\therefore \ln y = -2$

$\therefore y = e^{-2}$

$\therefore$ the limit is $\underline{\underline{e^{-2}}}$

Value	
3	10. Using the definition of derivative find $f'(x)$ if $f(x) = \sqrt{2x-5}$ and state where $f(x)$ is differentiable.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2x+2h-5} - \sqrt{2x-5}}{h} \cdot \frac{\sqrt{2x+2h-5} + \sqrt{2x-5}}{\sqrt{2x+2h-5} + \sqrt{2x-5}}$$

$$= \lim_{h \to 0} \frac{(2x+2h-5) - (2x-5)}{h\left[\sqrt{2x+2h-5} + \sqrt{2x-5}\right]}$$

$$= \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}\left[\sqrt{2x+2h-5} + \sqrt{2x-5}\right]}$$

$$= \lim_{h \to 0} \frac{2}{\sqrt{2x+2h-5} + \sqrt{2x-5}} = \frac{2}{2\sqrt{2x-5}}$$

$$f'(x) = \frac{1}{\sqrt{2x-5}}$$

f(x) is differentiable on

$\underline{x > 5/2}$

Value

1 11. (a) State what it means for a function f to be continuous at a.

If $f(x)$ is continuous at $x=a$, then

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$$

3 (b) Find the values of a and b so that

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if} \quad x < 2 \\ ax^2 - bx + 3 & \text{if} \quad 2 \le x < 3 \\ 2x - a + b & \text{if} \quad x \ge 3 \end{cases}$$

is continuous on $(-\infty, \infty)$.

@ $x=2$

$$\lim_{x\to 2^-} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = 4$$

$$\lim_{x\to 2^+} ax^2-bx+3 = 4a-2b+3$$

$$f(2) = 4a-2b+3$$

$\therefore$ $4a-2b+3=4$

$4a-2b-1=0$

$\therefore$ $10a-4b-3=0$

$8a-4b-2=0$

$2a \quad -1=0$

$\therefore$ $a=\frac{1}{2}$

@ $x=3$

$$\lim_{x\to 3^-} ax^2-bx+3 = 9a-3b+3$$

$$\lim_{x\to 3^+} 2x-a+b = 6-a+b$$

$$f(3) = 6-a+b$$

$9a-3b+3=6-a+b$

$10a-4b-3=0$

$10(\frac{1}{2})-4b-3=0$

$2=4b$

$\therefore$ $b=\frac{1}{2}$

Value

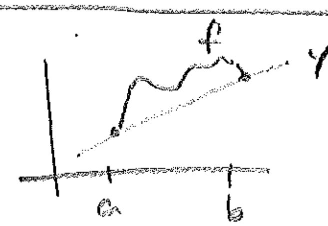
1 12. (a) State Rolle's Theorem for Derivatives.

If $f(x)$ is continuous on $[a,b]$
and $f'(x)$ exists on (a,b)
and $f(a) = f(b)$
then there is a pt $c \in (a,b)$ such that $f'(c) = 0$

4 (b) State and prove the Mean Value Theorem for derivatives.

If $f(x)$ is continuous on $[a,b]$ and $f'(x)$ exists on (a,b) then there is a pt $c \in (a,b)$ such that

1 $$f'(c) = \frac{f(b) - f(a)}{b-a}$$



Proof: let y pass through $(a, f(a))$, $(b, f(b))$

then $y = \dfrac{f(b) - f(a)}{b-a}(x-a) + f(a)$

3 Let $h(x) = f(x) - y(x)$. Since y is a straight line and therefore continuous and differentiable on $[a,b]$, then $h(x)$ satisfies Rolle's theorem. (Note: $h(a) = h(b) = 0$)

$\therefore$ By Rolle's theorem, there is a pt $c \in (a,b)$ such that $h'(c) = 0$.

But $h'(c) = f'(c) - y'(c) = 0$

$$\therefore f'(c) = y'(c) = \frac{f(b) - f(a)}{b-a}$$

Value

2 13. (a) State both parts of the Fundamental Theorem of Calculus (FTC 1 and FTC 2).

1) If $f(x)$ is continuous on $[a,b]$ and $x \in [a,b]$

1 and if $g(x) = \int_a^x f(t)\,dt$,

then $g'(x) = f(x)$

2) If $F(x)$ is an antiderivative of $f(x)$

1 then $\int_a^b f(x)\,dx = F(b) - F(a)$

3 (b) Define $\ln(x)$ and using your definition prove $\ln(xy) = \ln(x) + \ln(y)$ for $x, y > 0$.

For $x > 0$, $\ln x = \int_1^x \frac{1}{t}\,dt$

By FTC Part 1 $\frac{d}{dx} \ln x = \frac{1}{x}$

also $\frac{d}{dx} \ln(xy) = \frac{d}{dx} \int_1^{xy} \frac{1}{t}\,dt = \frac{1}{xy}(y) = \frac{1}{x}$

$\therefore \ln(xy) = \ln x + C$

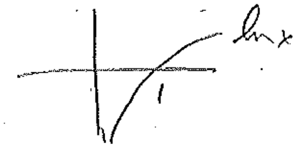
However $\ln(1) = \int_1^1 \frac{1}{t}\,dt = 0$

$\therefore \ln(1y) = \ln(1) + C \quad \therefore C = \ln y$

$\therefore \ln xy = \ln x + \ln y.$

Value	
3	14. Determine whether the following improper integral is convergent or divergent. If it is convergent, evaluate the integral.

$$\int_0^1 \ln x \, dx$$



$$\int_0^1 \ln x \, dx = \lim_{t \to 0^+} \int_t^1 \ln x \, dx$$

$$\int \ln x \, dx = x \ln x - x$$

$$= \lim_{t \to 0^+} (x \ln x - x)\Big|_t^1 = \lim_{t \to 0^+} (1 \ln 1 - 1) - (t \ln t - t)$$

$$= -1 - \lim_{t \to 0^+} (t \ln t - t)$$

$$= -1 - [0 - 0] = -1$$

$$\lim_{t \to 0^+} t \ln t = \lim_{t \to 0^+} \frac{\ln t}{\frac{1}{t}} = \lim_{t \to 0^+} \frac{\frac{1}{t}}{-\frac{1}{t^2}} = \lim_{t \to 0^+} -t = 0$$

$$\therefore \int_0^1 \ln x \, dx = \boxed{-1}$$

convergent

3	15. Find the arc length of the curve $y = \ln(\sec x)$ from $x = 0$ to $x = \frac{\pi}{4}$.

$$\frac{dy}{dx} = \frac{1}{\sec x} \sec x \tan x = \tan x$$

$$L = \int_0^{\pi/4} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$L = \int_0^{\pi/4} \sqrt{1 + \tan^2 x} \, dx$$

$$= \int_0^{\pi/4} \sec x \, dx$$

$$L = \ln|\sec x + \tan x| \Big]_0^{\pi/4}$$

$$= \ln(\sqrt{2} + 1) - \ln(1 + 0)$$

$$= \boxed{\ln(\sqrt{2} + 1)}$$

Total
100